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# One-dimensional drift–flux model for two-phase flow

in a large diameter pipe

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#### Abstract

In view of the practical importance of the drift–flux model for two-phase-flow analysis in general and in the analysis of nuclear-reactor transients and accidents in particular, the distribution parameter and the drift velocity have been studied for vertical upward two-phase flow in a large diameter pipe. One of the important flow characteristics in a large diameter pipe is a liquid recirculation induced at low mixture volumetric flux. Since the liquid recirculation may affect the liquid velocity profile and promote the formation of cap or slug bubbles, the distribution parameter and the drift velocity in a large diameter pipe can be quite different from those in a small diameter pipe where the liquid recirculation may not be significant. A flow regime at a test section inlet may also affect the liquid recirculation pattern, resulting in the inlet-flow-regime dependent distribution parameter and drift velocity. Based on the above detailed discussions, two types of inlet-flow-regime dependent drift–flux correlations have been developed for two-phase flow in a large diameter pipe at low mixture volumetric flux. A comparison of the newly developed correlations with various data at low mixture volumetric flux shows a satisfactory agreement. As the drift–flux correlations in a large diameter pipe at high mixture volumetric flux, the drift–flux correlations developed by Kataoka–Ishii, and Ishii have been recommended for cap bubbly flow, and churn and annular flows, respectively, based on the comparisons of the correlations with existing experimental data.

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#### 1. Introduction

Two-phase flows always involve some relative motion of one phase with respect to the other; therefore, a twophase-flow problem should be formulated in terms of two velocity fields. A general transient two-phase-flow problem can be formulated by using a two-fluid model [1,2] or a drift–flux model [3,4], depending on the degree

of the dynamic coupling between the phases. The drift– flux model is an approximate formulation in comparison with the more rigorous two-fluid formulation. However, because of its simplicity and applicability to a wide range of two-phase-flow problems of practical interest, the drift–flux model is of considerable importance. In view of the practical importance of the drift–flux model for two-phase-flow analysis, the drift–flux model has been studied extensively. In the state-of-the-art, the constitutive equations for the drift–flux model have been developed well for vertical upward two-phase flows in relatively small diameter pipes (25–50 mm) under relatively high flow rate conditions [5]. The constitutive equations obtained under the conditions have been often used in computational thermal–hydraulic codes. The

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constitutive equations given by Zuber and Findley [3], or Ishii [4] have been used in the present system codes such as TRAC-P1A, CANAC-II, and ATHOS 3 [5].

However, although large diameter piping systems are often encountered in nuclear power plants, the applicability of such constitutive equations to two-phase flow in a large diameter pipe has not been assured. In a large diameter pipe, slug bubbles cannot be sustained due to the interfacial instability and they disintegrate to cap bubbles. A recirculation flow pattern may develop in a large diameter pipe at low flow rate. A flow regime at a test section inlet and a flow regime transition in a developing flow may also have an influence on the liquid recirculation pattern. The liquid recirculation, inlet flow regime and flow regime transition may affect the transverse velocity and the void fraction profile significantly. The distribution parameter and the drift velocity in a large diameter pipe can be quite different from those in a small diameter pipe where the liquid recirculation may not be significant. Therefore, the effect of the flow channel size on the drift–flux model should be carefully examined in detail. In view of the practical importance, a few analytical and experimental studies related to twophase flow in a large diameter pipe have been performed for the past three decades. However, in the current status of this subject, insufficient systematic data bases are available to understand two-phase-flow characteristics in a large diameter pipe, and a reliable drift–flux model applicable to wide ranges of two-phase flow in a large diameter pipe has not been developed.

From this point of view, this study is aiming at a comprehensive literature survey to summarize the current understanding of the two-phase-flow characteristics in a large diameter pipe and the development of the drift–flux model for vertical upward two-phase flow in a large diameter pipe. The obtained drift–flux model is compared with existing experimental data taken under various experimental conditions such as flow channel diameters (0.102–0.480 m), pipe length-to-diameter ratio (4.2–108), pressures (0.1–1.5 MPa), mixture volumetric fluxes (0.03–6.1 m/s), bubble injection methods (test pipe with or without a horizontal section), and fluid systems (air–water, nitrogen–water, and steam–water).

# 2. Previous analytical and experimental works

2.1. One-dimensional drift–flux model and constitutive equations for distribution parameter and drift velocity in a small diameter pipe

The drift–flux model is one of the most practical and accurate models for two-phase-flow analysis. The model takes into account the relative motion between phases by a constitutive relation. It has been utilized to solve many engineering problems involving two-phase-flow

Nomenclature

dynamics [5]. In particular, its application to forced convection systems has been quite successful. The onedimensional drift–flux model is given as

$$
\frac{\langle j_g \rangle}{\langle \alpha \rangle} = \langle \langle v_g \rangle \rangle = C_0 \langle j \rangle + V_{\rm gi}, \tag{1}
$$

where  $j_g$ ,  $\alpha$ ,  $v_g$  and  $j$  are the superficial gas velocity, the void fraction, the gas velocity, and the mixture volumetric flux, respectively.  $\langle \rangle$  and  $\langle \langle \rangle \rangle$  mean the areaaveraged quantity over cross-sectional flow area and the void-fraction-weighted mean quantity, respectively. The distribution parameter,  $C_0$  and the drift velocity,  $V_{gi}$  are given as Eqs. (1) and (2), respectively.

$$
C_0 \equiv \frac{\langle \alpha j \rangle}{\langle \alpha \rangle \langle j \rangle},\tag{2}
$$

$$
V_{\rm gi} \equiv \frac{\langle v_{\rm gi} \alpha \rangle}{\langle \alpha \rangle},\tag{3}
$$

where  $v_{\rm gi}$  is the local drift velocity of a gas phase defined as  $v_{\text{gj}} = v_{\text{g}} - j$ .

Ishii [4] developed a simple correlation for the distribution parameter in upward two-phase flow. Ishii first considered a fully developed bubbly flow and assumed that the distribution parameter would depend on the density ratio,  $\rho_{\rm g}/\rho_{\rm f}$  and on the Reynolds number, Re. As the density ratio approaches the unity, the distribution parameter should become unity. Based on the limit and various experimental data in fully developed flows, the distribution parameter was given approximately by

$$
C_0 = C_{\infty}(Re) - \left\{ C_{\infty}(Re) - 1 \right\} \sqrt{\rho_g/\rho_f}, \tag{4}
$$

where  $C_{\infty}$  is the asymptotic value of  $C_0$  Here, the density group scales the inertia effects of each phase in a transverse void distribution. Physically, Eq. (4) models the tendency of the lighter phase to migrate into a higher velocity region, thus resulting in a higher void concentration in the central region [4]. Based on a wide range of Reynolds number, Ishii approximated  $C_{\infty}$  to be 1.2 for an upward flow in a round pipe [4]. Thus, for a fully developed turbulent flow in a round tube,

$$
C_0 = 1.2 - 0.2\sqrt{\rho_g/\rho_f}.\tag{5}
$$

Recently, Hibiki and Ishii [6] modified the constitutive equation of the distribution parameter for vertical upward bubbly flow based on a detailed discussion about bubble dynamics as

$$
C_0 = \left(1.2 - 0.2\sqrt{\rho_g/\rho_f}\right) \{1 - \exp(-22\langle D_{\rm Sm} \rangle/D)\},\quad (6)
$$

where  $D_{\rm Sm}$  and D are the Sauter mean diameter and the pipe diameter, respectively. This modified distribution parameter suggests that the dominant factor to determine the distribution parameter in vertical upward bubbly flow would be the bubble diameter.

Ishii [4] also developed the constitutive equation of the distribution parameter for boiling flow based on a detailed discussion on the effect of nucleate bubbles on a void distribution as

$$
C_0 = \left(1.2 - 0.2\sqrt{\rho_g/\rho_f}\right) \{1 - \exp(-18\langle \alpha \rangle)\}.
$$
 (7)

This modified distribution parameter suggests that the dominant factor to determine the distribution parameter in boiling flow would be the void fraction. Thus, a key to develop the constitutive equation of the distribution parameter is to find a dominant factor to determine the distribution parameter.

Ishii [4] also studied the kinematic constitutive equation for the drift velocity in various two-phase-flow regimes. The constitutive equation that specifies the relative motion between phases in the drift–flux model has been derived by taking into account the interfacial geometry, the body–force field, the shear stresses, and the interfacial momentum transfer, since these macroscopic effects govern the relative velocity between phases. Ishii [4] gave the following equations as drift-velocity correlations in various two-phase-flow regimes.

Bubbly flow

$$
V_{\rm gj} = \sqrt{2} \left( \frac{g \sigma \Delta \rho}{\rho_{\rm f}^2} \right)^{1/4} \left( 1 - \langle \alpha \rangle \right)^{1.75} \quad \text{for } \mu_{\rm f} \gg \mu_{\rm g}, \tag{8}
$$

where g,  $\sigma$ ,  $\Delta \rho$ ,  $\mu_f$  and  $\mu_g$  are the gravitational acceleration, the surface tension, the density difference between phases, the liquid viscosity and the gas viscosity, respectively.

Slug flow

$$
V_{\rm gj} = 0.35 \sqrt{\frac{g D \Delta \rho}{\rho_{\rm f}}}.\tag{9}
$$

Churn flow

$$
V_{\rm gj} = \sqrt{2} \left( \frac{g \sigma \Delta \rho}{\rho_{\rm f}^2} \right)^{1/4}.
$$
 (10)

Annular flow

$$
\overline{V}_{\rm gj} = \frac{1 - \langle \alpha \rangle}{\langle \alpha \rangle + \left\{ \frac{1 + 75(1 - \langle \alpha \rangle)}{\sqrt{\langle \alpha \rangle}} \frac{\rho_{\rm g}}{\rho_{\rm f}} \right\}^{1/2}} \left\{ \langle j \rangle + \sqrt{\frac{\Delta \rho \, g D (1 - \langle \alpha \rangle)}{0.015 \rho_{\rm f}}} \right\},\tag{11}
$$

where  $\overline{V}_{gi}$  is the mean transport drift velocity defined by  $V_{\rm gi} + (C_0 - 1)\langle j \rangle$  [4].

It is well-known that the constitutive equations explained here give excellent predictions for vertical upward two-phase flow in relatively small diameter (25–50 mm) pipes.

## 2.2. Existing drift–flux type correlations in a large diameter pipe

In what follows, some important studies on the development of the drift–flux type correlation for vertical upward two-phase flow in a large diameter pipe will be reviewed briefly.

Hills [7] measured void fraction in an air–water bubble column with an inner diameter of 0.150 m and height of 10.5 m at gas superficial velocities of 0.070–3.5 m/s and liquid superficial velocities of 0–2.7 m/s. Hills developed the following drift–flux type correlations based on his own data base:

$$
\langle \langle v_{\rm g} \rangle \rangle = 1.35 \langle j \rangle^{0.93} + 0.24, \quad \text{for } \langle j_{\rm f} \rangle > 0.3 \text{ m/s}, \qquad (12)
$$

$$
\frac{\langle j_g \rangle}{\langle \alpha \rangle} - \frac{\langle j_f \rangle}{1 - \langle \alpha \rangle} = 0.24 + 4.0 \langle \alpha \rangle^{1.72}, \quad \text{for } \langle j_f \rangle \leq 0.3 \text{ m/s}, \tag{13}
$$

where  $j_f$  is the superficial liquid velocity. In these correlations, the unit of parameters should be m/s. Since the mixture volumetric flux in his experiment should be 6.2 m/s at maximum, Eqs. (12) and (13) can be recast as Eqs.  $(12')$  and  $(13')$ , respectively.

$$
\langle \langle v_{\rm g} \rangle \rangle = 1.2 \langle j \rangle + 0.24, \quad \text{for } \langle j_{\rm f} \rangle > 0.3 \text{ m/s}, \tag{12'}
$$

$$
\langle \langle v_{\rm g} \rangle \rangle = \langle j \rangle + (4.0 \langle \alpha \rangle^{1.72} + 0.24)(1 - \langle \alpha \rangle),
$$
  
for  $\langle j_{\rm f} \rangle \leq 0.3$  m/s. (13')

It should be noted here that Hills did not consider the effect of physical properties on the distribution parameter and the drift velocity in his correlation. Thus, the applicability of Hills' correlation to other fluid systems such as high pressure steam–water flow is still questionable.

Shipley [8] measured void fraction of air–water bubbly flow in a pipe with an inner diameter of 0.457 m and height of 5.64 m. Shipley proposed the following correlation based on his own data base:

$$
\langle \langle v_{\rm g} \rangle \rangle = 1.2 \langle j \rangle + \left\{ 0.24 + 0.35 \left( \frac{\langle j_{\rm g} \rangle}{\langle j \rangle} \right)^2 \sqrt{g D \langle \alpha \rangle} \right\}.
$$
 (14)

In this correlation, the unit of parameters should be m/s. It should be noted here that the second term in the right hand side of this correlation corresponding to the drift velocity can become very large for a very large diameter pipe. This may not be sound physically.

Clark and Flemmer [9] measured void fraction of air– water bubbly flow in a pipe with an inner diameter of 0.10 m. Mixture volumetric fluxes and void fractions ranged from 0.7 to 2.7 m/s and from 0.05 to 0.25, respectively. They observed that the bubble sizes ranged from 1.5 to 5 mm in diameter and occasional large cap bubbles were formed. Clark and Flemmer proposed the following correlation based on their own data base:

$$
\langle \langle v_{\rm g} \rangle \rangle = 0.934(1 + 1.42 \langle \alpha \rangle) \langle j \rangle + 1.53 \left( \frac{\sigma g}{\rho_{\rm f}} \right)^{1/4}.
$$
 (15)

This correlation indicates that the distribution parameter increases from 0.934 to 1.33 at elevated void fractions from 0 to 0.3. Clark and Flemmer [10] also developed the following modified drift–flux type correlation as:

$$
\langle \langle v_{\rm g} \rangle \rangle = C_{\rm g} \langle j_{\rm g} \rangle + C_{\rm f} \langle j_{\rm f} \rangle + 0.25, \tag{16}
$$

where  $C_{\rm g}$  (=1.95) and  $C_{\rm f}$  (=0.93) are the profile constants as defined by

$$
C_{\rm g} = \frac{\langle j_{\rm g} \alpha \rangle}{\langle j_{\rm g} \rangle \langle \alpha \rangle} \quad \text{and} \quad C_{\rm f} = \frac{\langle j_{\rm f} \alpha \rangle}{\langle j_{\rm f} \rangle \langle \alpha \rangle}.
$$
 (17)

In this correlation, the unit of parameters should be m/s. It should be noted here that Clark and Flemmer did not consider the effect of physical properties on the distribution parameter and the drift velocity in their correlation. Thus, the applicability of Clark–Flemmer's correlation to other fluid systems such as high pressure steam–water flow is still questionable.

Hirao et al. [11,12] measured void fraction of steam– water two-phase flow using a large scale apparatus with 0.102 m diameter. Flow conditions were  $\langle j_f \rangle$  < 1 m/s and  $\langle j_g \rangle$  < 4 m/s. They classified the upward flow region into two regions in terms of the mixture volumetric flux such as 0 m/s  $\leq$  (*j*)  $\leq$  0.24 m/s and (*j*)  $>$  0.24 m/s. For  $\langle j \rangle > 0.24$  m/s, they developed the following correlation for the drift velocity as:

$$
V_{\rm gj} = 0.52 \sqrt{\frac{g D \Delta \rho}{\rho_{\rm f}}}.\tag{18}
$$

In the region of 0 m/s  $\leq \langle j \rangle \leq 0.24$  m/s, they proposed the interpolation between values calculated by Eq. (10) at  $\langle i \rangle = 0$  m/s and Eq. (18) at  $\langle i \rangle = 0.24$  m/s to obtain the drift velocity. In the correlation of Hirao et al., Eq. (5) was recommended as the distribution parameter. It should be noted here that the drift velocity in this correlation can become very large for a very large diameter pipe. This may not be sound physically.

Ishii and Kocamustafaogullari [13] developed a theoretical correlation of the drift velocity for cap bubble flow inside a large diameter channel. It is given by

$$
V_{\rm gj} = 0.54 \sqrt{\frac{g D_{\rm H} \Delta \rho}{\rho_{\rm f}}}, \quad \text{for } D_{\rm H}^* \leq 30,
$$
 (19)

$$
V_{\rm gi} = 3.0 \left( \frac{\sigma g \,\Delta \rho}{\rho_{\rm f}^2} \right)^{1/4}, \quad \text{for } D_{\rm H}^* \leqslant 30,
$$
 (20)

where  $D_H$  is the hydraulic diameter. The non-dimensional hydraulic diameter,  $D_{\rm H}^*$  is defined by

$$
D_{\rm H}^* = \frac{D_{\rm H}}{\sqrt{\sigma/g\,\Delta\rho}}.\tag{21}
$$

Equations (19) and (20) suggest that the drift velocity increases with the channel diameter and reaches to a constant value depending on physical properties at  $D_{\rm H}^* = 30$  corresponding to  $D_{\rm H} = 0.09$  m for air-water at atmospheric pressure. Equations (19) and (20) were derived under the assumption that the surface of the cap bubble was smooth. In real two-phase flow, large bubbles can be highly deformed due to natural turbulences in two-phase flow. However, it is noteworthy that the large bubbles in larger diameter channels approximately behave like a cap bubble rather than a slug bubble in terms of relative motion between phases. For channels with a diameter much larger than  $40\sqrt{\sigma/g\Delta\rho}$  corresponding to 0.1 m for air–water at atmospheric pressure, the slug bubbles cannot be sustained due to the surface instability and they are disintegrated into cap bubbles [13].

Kataoka and Ishii [14] found that the drift velocity in a pool system depended upon vessel diameter, system pressure, gas flux and fluid physical properties, and developed the following correlation for the pool void fraction based on extensive data bases taken under various experimental conditions:

Low viscous case:  $N_{\mu f} \leqslant 2.25 \times 10^{-3}$ 

$$
V_{\rm gj}^{+} = 0.0019 D_{\rm H}^{*^{0.809}} \left(\frac{\rho_{\rm g}}{\rho_{\rm f}}\right)^{-0.157} N_{\mu f}^{-0.562}, \quad \text{for } D_{\rm H}^{*} \leq 30,
$$
\n(22)

$$
V_{\rm gj}^{+} = 0.030 \left(\frac{\rho_{\rm g}}{\rho_{\rm f}}\right)^{-0.157} N_{\mu f}^{-0.562}, \quad \text{for } D_{\rm H}^{*} \geq 30. \tag{23}
$$

Higher viscous case:  $N_{\mu f} > 2.25 \times 10^{-3}$ 

$$
V_{\rm gj}^{+} = 0.92 \left(\frac{\rho_{\rm g}}{\rho_{\rm f}}\right)^{-0.157}, \quad \text{for } D_{\rm H}^{*} \geq 30,
$$
 (24)

where  $V_{\text{gi}}^+$  and  $N_{\mu f}$  are the non-dimensional drift velocity and the viscous number, respectively, defined as

$$
V_{\text{gj}}^{+} = \frac{V_{\text{gj}}}{\left(\frac{\sigma_{\text{g}}\Delta\rho}{\rho_{\text{f}}^{2}}\right)^{1/4}} \quad \text{and} \quad N_{\mu f} = \frac{\mu f}{\left(\rho_{\text{f}}\sigma\sqrt{\frac{\sigma}{g\Delta\rho}}\right)^{1/2}}.\tag{25}
$$

In Kataoka–Ishiis correlation, Eq. (5) was recommended to obtain the distribution parameter.

#### 2.3. Existing experimental works in a large diameter pipe

Hills [7] measured void fraction in an air–water bubble column with an inner diameter of 0.150 m and height of 10.5 m at gas superficial velocities of 0.07–3.5 m/s and liquid superficial velocities of 0–2.7 m/s.

Van der Welle [15] performed experiments in atmospheric vertical air–water flows, for void fractions between 0.25 and 0.75 and superficial liquid velocities of 1.3, 1.7 and 2.1 m/s. The local flow parameters measured by a resistivity probe technique included void fraction and bubble velocity as well as the bubble diameter. The measured data for void fraction and bubble velocity were correlated by means of power law distribution functions, with exponents given by a function of the cross-sectionally averaged void fraction.

Clark and Flemmer [9] measured void fraction of air– water bubbly flow in a pipe with an inner diameter of 0.10 m. Mixture volumetric fluxes and void fractions ranged from 0.7 to 2.7 m/s and from 0.05 to 0.25, respectively. They observed that the bubble sizes ranged from 1.5 to 5 mm in diameter and occasional large cap bubbles were formed.

Hirao et al. [11,12] carried out the experiments of cocurrent and countercurrent steam–water two-phase flows using a large scale apparatus with 0.102 m diameter pipe, and small scale apparatus with 0.0197 m diameter pipe. They discussed the pipe diameter effects on the drift–flux parameters such as the distribution parameter and the drift velocity based on their obtained data.

Hashemi et al. [16] investigated the effect of diameter and geometry on two-phase-flow regimes and carry-over in a model PWR hot leg. Void fraction measurement was conducted in a test rig with 0.305 m diameter pipe. Test conditions were selected to cover a wide range of gas and liquid superficial velocities expected to occur in a prototypical reactor geometry during a small break loss of coolant accident.

Onuki and Akimoto [17] investigated the flow structure of a developing air–water bubbly flow in a large diameter pipe with an inner diameter of 0.480 m and height of 2.0 m at superficial gas velocities of 0.02– 0.87 m/s and superficial liquid velocities of 0.01–0.2 m/s. In their experiment, two air injection methods (porous sinter injection and nozzle injection) were used to create an extremely different flow structure in the developing region. They reported that no slug bubbles occupying the flow path were observed regardless of the air injection methods even at the bubbly-to-slug flow transition region. They also compared the area-averaged void fraction near the top of the test section with Kataoka– Ishiis correlation and suggested that the distribution parameter of the drift–flux model should be remodeled by considering the effect of the pipe size. Onuki and Akimoto [18] also studied the transition characteristics of flow pattern and phase distribution of upward air– water flow along a large vertical pipe with an inner diameter of 0.200 m and the height of 12.3 m. The experiments were conducted at superficial gas velocities of 0.03–4.7 m/s and superficial liquid velocities of 0.06–1.06 m/s. They concluded the following remarks as the scale effect: (1) under low superficial liquid velocity where small-scale pipes would have a wall-peak phase distribution, some large eddies including bubble clusters filled

up the pipe and a core-peak phase distribution was observed, (2) the large coalescent bubbles were developed along the test section via the churn bubbly flow where the phase distribution was a core peak one, whereas Taylor bubbles in small-scale pipes were generated at the vicinity of gas–liquid mixing region or were developed from the bubbly flow with a wall-peak phase distribution, (3) the wall-peak in the large vertical pipe was lower even under the same bubble size. The lower peak could be associated with the lower radial velocity gradient of water and the larger turbulent dispersion force.

Hasanein et al. [19] conducted an experimental study on steam–water two-phase flow in a large diameter vertical piping at high pressures and temperatures. In their study, experimental data on steam–water twophase flow in a large diameter vertical pipe with an inner diameter of 0.508 m at elevated pressures and temperatures (2.8 MPa/230  $\degree$ C-6.4 MPa/280  $\degree$ C) were obtained. The averaged void fraction data were correlated in the form of a drift–flux correlation. The correlated data were then compared with Kataoka–Ishii's correlation.

Yoneda et al. [20] measured radial distributions of the flow structure of upward steam–water two-phase flow in a vertical pipe with an inner diameter of 0.155 m at superficial gas velocities smaller than 0.25 m/s and superficial liquid velocities smaller than 0.6 m/s by means of optical dual void probes. They observed that the flow would reach a quasi-developed state within relatively short height to diameter ratio  $( = 4)$  compared to a small-diameter pipe.

Hibiki and Ishii [21,22] performed experiments on hot-leg U-bend two-phase natural circulation in a loop with a large diameter pipe with an inner diameter of 0.102 m to understand the two-phase natural circulation and flow termination during a small break loss of coolant accident in light water reactors. They carried out various tests to understand the basic mechanism of the flow termination as well as to obtain essential information on scale effects of various parameters such as the loop frictional resistance, thermal center and pipe diameter. Measured flow parameters included the void distribution in a hot-leg, flow regime and natural circulation rate. They found that the formation of cap bubbles in the large diameter pipe caused the increased drift velocity, which would affect the prediction of the void fraction in the hot leg.

Shoukri et al. [23] examined the effect of pipe diameter on flow regime transition and void fraction in air– water flow in large diameter vertical pipes with inner diameters of 0.10 and 0.20 m. They measured radial distribution of void fraction by means of a local optical probe. They found that the transition from bubbly to intermittent flow was dependent on the pipe diameter. They correlated the area-averaged void fraction data by using the drift–flux model.

Smith et al. [24] measured axial development of flow parameters of bubbly flow in large diameter pipes with inner diameters of 0.102 and 0.152 m by means of four sensor conductivity probes. Measured flow parameters included void fraction, interfacial area concentration, and interfacial velocity. These data were used to evaluate the one-dimensional interfacial area transport equation.

## 3. Results and discussion

# 3.1. Database used to develop drift–flux correlation for two-phase flow in a large diameter pipe

A liquid recirculation flow pattern may develop in a large diameter pipe at low flow rate and the pattern would be governed mainly by the combination of gas and liquid superficial velocities. The developed recirculation flow pattern would affect the distribution parameter significantly, and the bubble coalescence enhanced by the liquid recirculation may create cap or slug bubbles resulting in the increase of the drift velocity. Thus, it is anticipated that the two-phase-flow data in a large diameter pipe at low flow rate may not collapse to a single line in the  $v_{\rm g} - j$  plane. Therefore, in order to develop the drift–flux correlation for two-phase flow in a large diameter pipe, many sets of superficial gas and liquid velocities and void fraction should be indispensable.

In relation to nuclear reactor accident and safety studies, the present authors performed experiments on hot-leg U-bend two-phase natural circulation in loops with inner diameters of 0.0508 and 0.102 m and height of 5.5 m to understand the two-phase natural circulation and flow termination during a small break loss of coolant accident in light water reactors [21,22,25]. The loop design was based on the scaling criteria developed under this program, and enough flexibility was built into the design such that various parametric effects and scale distortions could be studied by changing some components. One of the important aspects was that a horizontal section could be inserted between the gas injector and the hot leg in order to investigate the effect of the gas phase inlet section on the flow regimes and flow interruption. Here, the gas injector consisted of 625 nozzles, which were made of stainless steel tubes, having a nominal 0.015 cm inner diameter and 0.03 cm outer diameter. Thus, when no horizontal section was inserted between the gas injector and the hot leg, the flow regime at the inlet was uniformly distributed bubbly flow, whereas when the horizontal section was inserted between them, the flow regime at the inlet was cap bubbly flow or slug flow at the void fraction higher than 0.1 due to the phase stratification in the horizontal section. The loop was operated either in a natural circulation mode or in a forced circulation mode using nitrogen gas and water. The flow measurements were performed at three axial locations,  $z/D = 12.8$ , 26.6 and 41.8, to investigate the effect of the flow development of the void fraction. A total of 59 and 12 data sets were acquired for the 0.102 mdiameter pipe without and with the horizontal section operated in the forced circulation mode, respectively [21,22]. A total of 73 data sets were also acquired for the 0.0508 m-diameter pipe without the horizontal section operated in the forced circulation mode [25]. The loop design and the experimental procedure were detailed in our previous papers [21,22,25].

In addition to our database, four databases [7,11,12,16,17] listed in Table 1 are also available. Since two databases acquired by Hills [7] and Hashemi et al. [16] include complete sets of superficial gas and liquid velocities and void fraction, they can be used for the development of the drift–flux correlation at low flow rate. Since two other databases acquired by Hirao et al. [11,12] and Ohnuki and Akimoto [17] only include sets of gas velocity and mixture volumetric flux, they may not be used for the development of the drift–flux correlation at low flow rate. However, the database acquired by Hirao et al. [11,12] is useful for testing the applicability of the developed drift–flux correlation to steam–water two-phase flow under elevated pressure conditions. The database acquired by Ohnuki and Akimoto [17] can also be utilized for discussing the effect of the gas injection method on flow parameters. These databases widely cover extensive experimental conditions such as flow channel diameters (0.102–0.480 m), pipe length-to-diameter ratio (4.2–108), pressures (0.1– 1.5 MPa), mixture volumetric fluxes (0.03–6.1 m/s), bubble injection methods (test pipe with or without a horizontal section), and fluid systems (air–water, nitrogen–water, and steam–water). The detailed experimental conditions are shown in Table 1. As a result, a total of 609 data sets are available to develop and to evaluate the drift–flux correlation for upward two-phase flow in a large diameter pipe.

#### 3.2. Comparison of existing drift–flux correlations with data taken at low flow rate

Various drift–flux correlations are compared with nitrogen–water flow data taken in vertical pipes  $(D = 0.0508$  and 0.102 m) with or without a horizontal section at low mixture volumetric flux. Here, following non-dimensional parameters are introduced to nondimensionalize the drift–flux model:

$$
\langle j_{\rm g}^{+} \rangle = \frac{\langle j_{\rm g} \rangle}{\left(\frac{\sigma_{\rm g} \Delta \rho}{\rho_{\rm f}^{2}}\right)^{1/4}}, \quad \langle j_{\rm f}^{+} \rangle = \frac{\langle j_{\rm f} \rangle}{\left(\frac{\sigma_{\rm g} \Delta \rho}{\rho_{\rm f}^{2}}\right)^{1/4}},
$$
  

$$
\langle j^{+} \rangle = \frac{\langle j \rangle}{\left(\frac{\sigma_{\rm g} \Delta \rho}{\rho_{\rm f}^{2}}\right)^{1/4}}, \quad \text{and} \quad \langle \langle v_{\rm g}^{+} \rangle \rangle = \frac{\langle \langle v_{\rm g} \rangle \rangle}{\left(\frac{\sigma_{\rm g} \Delta \rho}{\rho_{\rm f}^{2}}\right)^{1/4}}.
$$
 (26)



Databases used in this study

The non-dimensionalized drift–flux model is expressed as:

$$
\frac{\langle j_g^+ \rangle}{\langle \alpha \rangle} = \langle \langle v_g^+ \rangle \rangle = C_0 \langle j^+ \rangle + V_{\text{gj}}^+.
$$
 (27)

The drift velocities calculated by various correlations are shown in Table 2. Data measured in the 0.0508 mdiameter pipe are compared with drift–flux correlations for bubbly, slug and churn flows given by Ishii [4] in Fig. 1. In Fig. 1, open and solid symbols indicate that observed flow regimes are bubbly and slug flows, respectively. Solid, broken and dotted lines mean the calculated values by Ishii's equations for bubbly flow at  $\langle \alpha \rangle = 0.3$ corresponding to the bubbly-to-slug flow transition boundary in a relatively small diameter pipe, for bubbly flow at  $\langle \alpha \rangle = 0$  or for churn flow, and for slug flow, respectively. The data taken in the 0.0508 m-diameter pipe almost collapse to a single line regardless of the flow regime. This may be explained by insignificant liquid recirculations and similar drift velocity among bubbly, slug and churn flows (see Table 2). The drift–flux correlations with the drift velocity given by Ishii can predict the data taken in the relatively small diameter pipe over all flow range tested in this experiment.

Data measured in the 0.102 m-diameter pipe without or with a horizontal section are compared with various drift–flux correlations in Fig. 2(a) or (b), respectively, as a parameter of superficial gas velocity. The meanings of calculated lines in Fig. 2(a) and (b) are given in the figures. Fig. 2(a) reveals the effect of the pipe diameter on the drift–flux plot. The data for bubbly flow in the 0.102 m-diameter pipe do not fall on a single line at low mixture volumetric flux where the bubble-induced turbulence would play an important role in determining the flow field. For different superficial gas velocities, the distribution parameter and the drift velocity vary between the correlations for bubbly flow and for cap bubbly flow. The drift velocity seems to be much higher





Fig. 1. Comparison of various drift–flux correlations with nitrogen–water data taken in a pipe with an inner diameter of 0.0508 m [21,25].

than the value predicted by the drift–flux correlation for bubbly flow. This can be explained as follows [26].

Higher drift velocity in a large diameter pipe may be due to the occurrence of some large cap bubbles in the center region of the pipe, although the flow looks like bubbly flow by visualization. These large cap bubbles move faster than the dispersed bubbles, resulting in a significant increase in the drift velocity. Because of the high void peak in the center region, where the gas bubbles drive the liquid to a much higher velocity than in the near-wall region. This phenomenon has been observed from the local measurement of interfacial area concentration in vertical bubbly flows [27–29]. It was identified as the so-called channeling effect, referring to the phenomenon that a fast moving center core accompanied by a slow moving out-layer or even occasional



The distribution parameters at  $P = 0.1$  and 1.5 MPa are 1.19 and 1.18, respectively. Values in parentheses indicate non-dimensionalized drift velocity,  $V_{\text{gi}}^{+}$ .

hj<sup>þ</sup>



Fig. 2. Comparison of various drift–flux correlations with nitrogen–water data taken in a pipe with an inner diameter of 0.102 m: (a) without a horizontal section [21] and (b) with a horizontal section [22].

recirculations near the wall region. Similar phenomenon (chimney effect) was identified in the rod bundles of a reactor core, where large cap bubbles rise in the center region and the coolant may circulate downward near the wall area, especially when there is a non-uniform power distribution in a bundle [30]. Such channeling effect would increase the distribution parameter and the drift velocity. However, this effect seems to be diminished at high flow rate where the shear-induced turbulence would play an important role in determining the flow field. This may be due to an insignificant recirculation flow pattern at high flow rate. Thus, as the flow rate is increased, the data tend to converge at their asymptotic values depending on their flow regimes such as bubbly, cap bubbly, slug and churn flows.

Fig. 2(b) reveals the effect of the inlet condition on the drift–flux plot. For the large diameter pipe  $(D = 0.102 \text{ m})$  without the horizontal section, the distribution parameter and the drift velocity appear to be dependent on the superficial gas velocity, whereas the data for the large diameter pipe ( $D = 0.102$  m) with the horizontal section almost collapse to a single line regardless of the superficial gas velocity. The behavior of the data may be explained by the flow regime at the inlet. For the large diameter pipe without the horizontal section, the flow regime at the inlet was bubbly flow, whereas the slug or cap bubbles were already formed at the inlet for the large diameter pipe with the horizontal section. Therefore, the flow characteristics in the large diameter pipe with the horizontal section is mainly dominated by the development of liquid recirculation pattern, whereas the flow characteristics in the large diameter pipe without the horizontal section is governed by the development of the liquid recirculation pattern and the flow regime transition from bubbly flow to cap bubbly or slug flow.

The correlation of Hirao et al. [11,12] cannot predict the data trend for the large diameter pipe without the horizontal section but for the large diameter pipe with the horizontal section. The correlation of Hirao et al. was developed based on their own data taking by using L-shaped and vertical pipes similar to pipes with and without the horizontal section in our experiment, respectively. Hirao et al. [11,12] observed no significant difference in the experimental results between the Lshaped and vertical pipes even at low mixture volumetric flux, which were quite different from our results. This might be due to the design of the gas injector in the experiment of Hirao et al., consisting of a sintered metal with many  $20$ -µm holes [11,12]. At low flow rate, it might be difficult to produce small dispersed bubbles using this gas injector. Thus, in the experiment of Hirao et al., the flow regime at the inlet might be cap bubbly or slug flow at low flow rate regardless of the inlet geometry. Therefore, the correlation of Hirao et al. developed at low mixture volumetric flux may be applicable to the inlet condition such as cap bubbly or slug flow.

Other existing drift–flux type correlations developed at low flow rate are also compared with the nitrogen– water data taken in the 0.102 m-diameter pipe without the horizontal section. Fig. 3 shows the comparison of Hills' correlation [7] with the data. The overall agreement between them may be acceptable. However, the correlation cannot reproduce the steep increase in the gas velocity against the mixture volumetric flux at low mixture volumetric flux and underestimates the data for  $\langle j_f^+ \rangle > 1.84$  corresponding to  $\langle j_f \rangle > 0.3$  m/s for nitrogen–water at atmospheric pressure. Figs. 4 and 5 show



Fig. 3. Comparison of Hills' correlation [7] with nitrogen–water data taken in a pipe with an inner diameter of 0.102 m without a horizontal section [21].



Fig. 4. Comparison of Shipley's correlation [8] with nitrogenwater data taken in a pipe with an inner diameter of 0.102 m without a horizontal section [21].

the comparison of Shipley's correlation [8] and Clark– Flemmer's correlation proposed in 1985 [9] with the data, respectively. Unfortunately, none of them can give satisfactory predictions. Fig. 6 compares Clark–Flemmer's correlation proposed in 1986 [10] with the data. The overall agreement between them is fairly good. However, for  $\langle j_g^+ \rangle < 0.4$  the correlation overestimates the gas velocity, whereas for  $0.5 < \langle j_g^+ \rangle < 1.2$  the correlation underestimates the gas velocity. It should be also pointed out that these correlations were developed



Fig. 5. Comparison of Clark–Flemmer's correlation [9] with nitrogen–water data taken in a pipe with an inner diameter of 0.102 m without a horizontal section [21].



Fig. 6. Comparison of Clark–Flemmer's correlation [10] with nitrogen–water data taken in a pipe with an inner diameter of 0.102 m without a horizontal section [21].

based on adiabatic air–water flow data. Since physical properties would affect the drift velocity significantly [14], the applicability of these correlations to highpressure or other fluid systems is questionable.

## 3.3. Development of drift–flux correlation for two-phase flow in a large diameter pipe at low flow rate

As discussed in the previous section, flow characteristics in a large diameter pipe would be influenced strongly by an inlet condition. Therefore, two types of drift–flux correlations for two-phase flow in a large diameter pipe at low flow rate may be necessary for inlet conditions such that (1) uniformly distributed bubbles are introduced into a test section, and (2) cap or slug bubbles are introduced into a test section. First, the drift–flux correlation for the inlet flow regime such as uniformly distributed bubbly flow is developed.

The void-fraction-weighted mean gas velocity,  $\langle j_g \rangle / \langle \alpha \rangle$ , and the cross-sectional mean mixture volumetric flux,  $\langle i \rangle$ , are easily obtainable parameters in experiments. Therefore, Eq. (1) suggests a plot of  $\langle j_g \rangle / \langle \alpha \rangle$ versus  $\langle j \rangle$ . An important characteristic of such a plot is that, for two-phase-flow regimes with fully developed void and velocity profiles, the data points cluster around a straight line. The value of the distribution parameter,  $C_0$  has been obtained indirectly from the slope of the line, whereas the intercept of this line with the voidfraction-weighted mean local drift velocity,  $V_{gi}$ . However, since two-phase flow in a large diameter pipe may not be fully developed at low flow rate, the distribution parameter and the drift velocity determined from such a plot may not reflect their true flow characteristics. As recent development of local sensor techniques [31,32] enables measurement of local flow parameters such as void fraction, and gas and liquid velocities, the values of  $C_0$  and  $V_{gi}$  in a bubbly flow can be determined directly by Eqs. (2) and (3) from experimental data of the local flow parameters. However, since sufficient data of local flow parameters in a large diameter pipe are not available, it is difficult to develop a detailed drift–flux correlation. Instead, an approximated drift–flux correlation, which can be applicable to two-phase flow in a large diameter pipe at low flow rate, is developed here.

As shown in Fig. 2(a), the drift–flux correlations given by Ishii [4], which can be applicable to wide ranges of two-phase flow in a relatively small diameter pipe, could predict the data taken in a forced convective flow well at  $\langle j_{\rm g}^+ \rangle \leq 0.5$ . Kataoka and Ishii [14] reported the similar observation such that Ishii's drift–flux correlations [4] could predict the experimental data taken in a pool at  $\langle j_g^+ \rangle \leq 0.5$ . The reason why Ishii's drift–flux correlations can be applicable to two-phase flow even in the large diameter pipe at  $\langle j_g^+ \rangle \leq 0.5$  would be due to relatively small liquid recirculations at  $\langle j_g^+ \rangle \leq 0.5$ . Thus, it is expected that the flow characteristics in a large diameter pipe at  $\langle j_g^+ \rangle \leq 0.5$  may be similar to that in a relatively small diameter pipe. However, as the gas superficial velocity increases, the drift velocity would increase due to the formation of cap bubbles. For relatively high  $\langle j_g^+ \rangle$  at low  $\langle j^+ \rangle$ , the liquid flux,  $\langle j_f^+ \rangle$  is sufficiently low and the experimental results approximately represent the phenomena in bubbling or pool boiling systems [14]. Thus, for high  $\langle j_{\rm g}^{\dagger} \rangle$  at low  $\langle j^{\dagger} \rangle$ , the drift velocity may be given by Kataoka–Ishiis correlation [14]. Taking account of the flow regime transition from bubbly flow to cap bubbly flow with the superficial gas velocity, the drift velocity in a large diameter pipe at low flow rate may be approximated by

$$
V_{\rm gj}^{+} = V_{\rm gj,B}^{+} \exp(A\langle j_{\rm g}^{+} \rangle) + V_{\rm gj,P}^{+} \{ 1 - \exp(A\langle j_{\rm g}^{+} \rangle) \},\tag{28}
$$

where the subscripts of B and P mean the drift-velocity correlations for bubbly flow and bubbling or pool boiling systems, respectively.  $V_{\text{gi},B}^+$  and  $V_{\text{gi},P}^+$  are given by Eq. (8) and Eqs. (22)–(24), respectively. The coefficient, A, may be roughly estimated to be  $-1.39$  by the condition of  $exp(A\langle j_{g}^{+}\rangle) = 0.5$  at  $\langle j_{g}^{+}\rangle = 0.5$ . Approximating the drift velocity to be Eq. (28), the distribution parameter is then determined by Eq. (1) or Eq. (27) with void fraction, and superficial gas and liquid velocities measured in our experiment using a 0.102 m-diameter pipe without the horizontal section [21]. It should be noted here that the prediction error in the drift velocity due to this approximation is imposed upon the estimation in the distribution parameter.

Fig. 7 indicates that the distribution parameter may correlate closely with the ratio of superficial gas velocity to mixture volumetric flux defined by  $\langle j_g^+ \rangle / \langle j^+ \rangle$ . The experimental result shows that the distribution parameter increases up to a certain value and gradually decreases as  $\langle j_g^+ \rangle / \langle j^+ \rangle$  increases. The distribution parameter is likely to have a threshold with respect to  $\langle j_g^+ \rangle / \langle j^+ \rangle$ , and this threshold may correspond to the transition point between enhancement and reduction of a liquid recirculation due to gas injection. In the ''enhancement" region, as the superficial gas velocity increases, the liquid recirculation flow pattern may gradually develop resulting in the gradual increase of the distribution parameter. However in the ''reduction'' region, further increase in the superficial gas velocity may hinder the liquid recirculation flow pattern. This is similar to ''flooding'' phenomena. This trend suggests the following functional form to correlate the asymptotic value of the distribution parameter for the inlet flow regime such as uniformly distributed bubbly flow in a large diameter pipe, since the distribution parameter is approximately equal to the asymptotic value of the distribution parameter for nitrogen–water system at atmospheric pressure, namely  $C_{\infty} \approx C_0$ .

$$
C_{\infty} = \exp\left\{a\left(\frac{\langle j_{\rm g}^{+}\rangle}{\langle j^{+}\rangle}\right)^{b}\right\},\
$$
  
for  $0 \le \langle j_{\rm g}^{+}\rangle/\langle j^{+}\rangle \le (\langle j_{\rm g}^{+}\rangle/\langle j^{+}\rangle)_{\rm tr},$   

$$
C_{\infty} = \frac{C_{\infty,\rm tr} - C_{\infty,1}}{(\langle j_{\rm g}^{+}\rangle/\langle j^{+}\rangle)_{\rm tr} - 1} \left(\frac{\langle j_{\rm g}^{+}\rangle}{\langle j^{+}\rangle}\right) + \frac{C_{\infty,\rm tr} - C_{\infty,1}(\langle j_{\rm g}^{+}\rangle/\langle j^{+}\rangle)_{\rm tr}}{1 - (\langle j_{\rm g}^{+}\rangle/\langle j^{+}\rangle)_{\rm tr}},\
$$
  
for  $(\langle j_{\rm g}^{+}\rangle/\langle j^{+}\rangle)_{\rm tr} \le \langle j_{\rm g}^{+}\rangle/\langle j^{+}\rangle \le 1$  (9)



Fig. 7. Dependence of distribution parameter in a large diameter pipe without a horizontal section on ratio of superficial gas velocity to mixture volumetric flux.

where  $(\langle j_{\rm g}^+ \rangle / \langle j^+ \rangle)_{\rm tr}$ ,  $C_{\infty,\rm tr}$  and  $C_{\infty,1}$  are the threshold value of  $\langle j^{\dagger}_{g} \rangle / \langle j^{\dagger} \rangle$  corresponding to the transition point between ''enhancement'' and ''reduction'' regions, the value of  $C_\infty$  at  $(\langle j_g^+ \rangle / \langle j^+ \rangle)_{tr}$ , and the value of  $C_\infty$  at  $\langle j_g^+ \rangle / \langle j^+ \rangle = 1$ respectively. *a* and *b* are adjustable parameters to be determined based on the data. Here, the threshold value of  $(\langle j_{\rm g}^+ \rangle / \langle j^+ \rangle)_{\rm tr}$ , may be approximated to be 0.9 based on the data graphically (see Fig. 7). The asymptotic value of the distribution parameter at  $\langle j_g^+ \rangle / \langle j^+ \rangle = 1$ ,  $C_{\infty,1}$ , may be assumed to be 1.2, which is the same as that for bubbling or pool boiling system. Then, the drift–flux correlation to be developed here becomes identical to Kataoka–Ishii's correlation for bubbling or pool boiling system at high  $\langle j_{\rm g}^+ \rangle$ . From these considerations, the constitutive equation for the distribution parameter for two-phase flow in a large diameter pipe with the inlet flow regime such as uniformly distributed bubbly flow is finalized by 59 data sets obtained in our previous experiment using a 0.102 mdiameter pipe without the horizontal section [21] by means of the least-square method as

$$
C_0 = \exp\left\{0.475\left(\frac{\langle j_{\rm g}^{+}\rangle}{\langle j^{+}\rangle}\right)^{1.69}\right\}
$$
  
\n
$$
-\left[\exp\left\{0.475\left(\frac{\langle j_{\rm g}^{+}\rangle}{\langle j^{+}\rangle}\right)^{1.69}\right\} - 1\right]\sqrt{\frac{\rho_{\rm g}}{\rho_{\rm f}}},
$$
  
\nfor  $0 \le \langle j_{\rm g}^{+}\rangle/\langle j^{+}\rangle \le 0.9,$   
\n
$$
C_0 = \left\{-2.88\left(\frac{\langle j_{\rm g}^{+}\rangle}{\langle j^{+}\rangle}\right) + 4.08\right\}
$$
  
\n
$$
-\left\{-2.88\left(\frac{\langle j_{\rm g}^{+}\rangle}{\langle j^{+}\rangle}\right) + 3.08\right\}\sqrt{\frac{\rho_{\rm g}}{\rho_{\rm f}}},
$$
  
\nfor  $0.9 \le \langle j_{\rm g}^{+}\rangle/\langle j^{+}\rangle \le 1,$  (30)

or

$$
C_0 = \exp\left\{0.475\left(\frac{\langle j_{\rm g}^+ \rangle}{\langle j^+ \rangle}\right)^{1.69}\right\}
$$
  
\n
$$
\times \left(1 - \sqrt{\frac{\rho_{\rm g}}{\rho_{\rm f}}}\right) + \sqrt{\frac{\rho_{\rm g}}{\rho_{\rm f}}}, \quad \text{for } 0 \le \langle j_{\rm g}^+ \rangle / \langle j^+ \rangle \le 0.9,
$$
  
\n
$$
C_0 = \left\{-2.88\left(\frac{\langle j_{\rm g}^+ \rangle}{\langle j^+ \rangle}\right) + 4.08\right\}
$$
  
\n
$$
\times \left(1 - \sqrt{\frac{\rho_{\rm g}}{\rho_{\rm f}}}\right) + \sqrt{\frac{\rho_{\rm g}}{\rho_{\rm f}}}, \quad \text{for } 0.9 \le \langle j_{\rm g}^+ \rangle / \langle j^+ \rangle \le 1.
$$
  
\n(31)

The solid line in Fig. 7 shows the distribution parameter calculated by Eq. (30). The newly developed constitutive equation for the distribution parameter, Eq. (30), gives reasonably good prediction for the distribution parameter over a whole range of  $\langle j_g^+ \rangle / \langle j^+ \rangle$ . It should be noted here that the distribution parameter near  $(\langle j_{\rm g}^{+} \rangle / \langle j^{+} \rangle)_{tr}$  shows the value higher than 1.2. Clark and Flemmer [33] numerically showed that the distribution parameter could be higher than 1.2 when the liquid recirculation developed. Hills [7] reported that the liquid flow direction was downward near the wall in a large diameter pipe. Recently, Ohnuki et al. [34] also observed that the direction of liquid flow near the wall was downward at the higher superficial gas velocity. Therefore, the reason why the distribution parameters near  $(\langle j_{g}^{+} \rangle / \langle j^{+} \rangle)_{tr}$  are higher than 1.2 can be explained by the liquid recirculation flow induced in the large diameter pipe and resulting high core peaking in the void fraction distribution.

Next, the drift–flux correlation for the inlet flow regime such as cap or slug bubbles is developed. Since the inlet flow regime is cap or slug bubbles, we approximate the drift velocity to be Kataoka–Ishiis correlation [14]. Approximating the drift velocity to be Kataoka–Ishii's correlation, the distribution parameter is then determined by Eq. (1) or Eq. (27) with void fraction, and superficial gas and liquid velocities measured in our experiment using a 0.102 m-diameter pipe with the horizontal section [22]. It should be noted here that the prediction error in the drift velocity due to this approximation is imposed upon the estimation in the distribution parameter.

Fig. 8 indicates that the distribution parameter may correlate closely with the non-dimensional mixture volumetric flux. The experimental result shows that the distribution parameter monotonically increases at low  $\langle i^{+} \rangle$ . However, it has been reported that the distribution parameter can be approximated to be 1.2 for high  $\langle j^+ \rangle$ [11,12,21]. This implies that the distribution parameter increases up to a certain value and decreases toward a certain value of the distribution parameter as the mixture volumetric flux increases. Thus, the following functional form to correlate the asymptotic value of the



Fig. 8. Dependence of distribution parameter in a large diameter pipe with a horizontal section on mixture volumetric flux.

distribution parameter for the inlet flow regime such as cap bubbly or slug flow in a large diameter pipe may be assumed, since the distribution parameter is approximately equal to the asymptotic value of the distribution parameter for nitrogen–water system at atmospheric pressure, namely  $C_{\infty} \approx C_0$ .

$$
C_{\infty} = C_{\infty,0} \exp(a'\langle j^{+}\rangle^{b'}),
$$
  
for  $0 \le \langle j^{+}\rangle \le \langle j^{+}\rangle_{\text{TR}},$   

$$
C_{\infty} = C_{\infty,\text{TR}} \exp\{a''(\langle j^{+}\rangle - b'')\}
$$
  
+ 
$$
C_{\infty,\infty}[1 - \exp\{a''(\langle j^{+}\rangle - b'')\}],
$$
  
for  $\langle j^{+}\rangle_{\text{TR}} \le \langle j^{+}\rangle$  (32)

where  $\langle j^+ \rangle_{TR}$ ,  $C_{\infty,0}$ ,  $C_{\infty,TR}$  and  $C_{\infty,\infty}$  are the threshold value of  $\langle j^+ \rangle$ , the values of  $C_\infty$  at  $\langle j^+ \rangle = 0$  and  $\langle j^+ \rangle_{TR}$ and the asymptotic value of  $C_{\infty}$  at very high  $\langle j^{+} \rangle$ , respectively.  $a'$ ,  $a''$ ,  $b'$  and  $b''$  are adjustable parameters to be determined based on the data. Here, the threshold value of the non-dimensional mixture volumetric flux,  $\langle j^{+} \rangle_{TR}$  may be approximated to be 1.8 based on the data graphically (see Fig. 8). The value of  $C_{\infty,0}$ , may be assumed to be 1.2, since large bubbles may rise near the pipe center resulting in core void peak and no liquid recirculation would be induced at  $\langle i^{+} \rangle = 0$ . In fact, even improper choice of  $C_{\infty,0}$  may not affect the prediction accuracy of the void fraction at very low  $\langle j^{+} \rangle$  where  $C_0\langle j^+\rangle \ll V_{\rm gi}^+$ . The value of  $C_{\infty,\infty}$ , may be assumed to be 1.2, since the void fraction distribution has core peak and the liquid recirculation would be suppressed at high  $\langle j^{+} \rangle$ . Then, at high  $\langle j^{+} \rangle$ , the drift–flux correlation to be developed here is identical to Kataoka–Ishii's correlation. Since sufficient data at high  $\langle i^{+} \rangle$  are not available, it is difficult to determine  $a''$ . However, Fig. 2 suggests that the drift–flux correlation converges to Kataoka– Ishii's correlation at  $\langle j^{+} \rangle$  higher than 5. Based on this observation,  $a''$  is determined. From these considerations, the constitutive equation for the distribution parameter for two-phase flow in a large diameter pipe with the inlet flow regime such as cap bubbly or slug flow is finalized by 12 data sets obtained in our previous experiment using a 0.102 m-diameter pipe with the horizontal section [22] by means of the least-square method as

$$
C_0 = 1.2 \exp(0.110 \langle j^+ \rangle^{2.22})
$$
  
\n
$$
- \{1.2 \exp(0.110 \langle j^+ \rangle^{2.22}) - 1\} \sqrt{\frac{\rho_g}{\rho_f}},
$$
  
\nfor  $0 \le \langle j^+ \rangle \le 1.8$ ,  
\n
$$
C_0 = [0.6 \exp\{-1.2(\langle j^+ \rangle - 1.8)\} + 1.2]
$$
  
\n
$$
- [0.6 \exp\{-1.2(\langle j^+ \rangle - 1.8)\} + 0.2] \sqrt{\frac{\rho_g}{\rho_f}},
$$
  
\nfor  $1.8 \le \langle j^+ \rangle$ . (33)

or

$$
C_0 = 1.2 \exp(0.110 \langle j^+ \rangle^{2.22})
$$
  
\n
$$
\times \left(1 - \sqrt{\frac{\rho_g}{\rho_f}}\right) + \sqrt{\frac{\rho_g}{\rho_f}}, \quad \text{for } 0 \le \langle j^+ \rangle \le 1.8,
$$
  
\n
$$
C_0 = [0.6 \exp\{-1.2(\langle j^+ \rangle - 1.8)\} + 1.2]
$$
  
\n
$$
\times \left(1 - \sqrt{\frac{\rho_g}{\rho_f}}\right) + \sqrt{\frac{\rho_g}{\rho_f}}, \quad \text{for } 1.8 \le \langle j^+ \rangle.
$$
 (34)

The solid line in Fig. 8 shows the distribution parameter calculated by Eq. (33). The newly developed constitutive equation for the distribution parameter (Eq. (33)) gives reasonably good prediction for the distribution parameter over a whole range of  $\langle i^{+} \rangle$ .

As discussed above, the key parameter determining the distribution parameter may be dependent on the inlet flow regime. For the large diameter pipe without the horizontal section, the flow regime at the inlet was bubbly flow, whereas the slug or cap bubbles were already formed at the inlet for the large diameter pipe with the horizontal section. Therefore, the flow characteristics in the large diameter pipe with the horizontal section is mainly dominated by the development of liquid recirculation pattern, whereas the flow characteristics in the large diameter pipe without the horizontal section is governed by the development of the liquid recirculation pattern and the flow regime transition from bubbly flow to cap bubbly or slug flow.

#### 3.4. Comparison of newly developed drift–flux correlations with experimental data

In this section, the newly developed drift–flux correlations for two-phase flow in a large diameter pipe at low flow rate are evaluated by existing data listed in Table 1.

First, the newly developed drift–flux correlation for the inlet flow regime such as uniformly distributed bubbly flow, Eq.  $(27)$  with Eqs.  $(28)$  and  $(30)$ , is compared with data taken by Hibiki and Ishii [21], Hills [7], and Hashemi et al. [16] in Figs. 9, 10 and 11, respectively. Solid lines in Fig. 9 indicate calculated values by the newly developed drift–flux correlation at average void fractions of each data group with the same superficial gas velocity. Solid and broken lines in the Figs. 10 and 11 indicate calculated values by the newly developed drift– flux correlation at  $\langle \alpha \rangle = 0$  and 0.3, respectively. Since Hills and Hashemi et al. carried out the experiments by changing the superficial gas velocity, keeping the superficial liquid velocity constant, the data in Figs. 10 and 11 are presented as a parameter of the superficial liquid velocity. The newly developed drift–flux model can predict the proper trend and the value of the experimental data very well. An average relative deviation between the newly developed drift–flux correlation and 373 data sets shown in Figs. 9–11 is estimated to be  $\pm 6.66\%$ . Fig. 12 also compares the newly developed drift–flux correlation with data taken by Ohnuki and Akimoto [17]. Unfortunately, since no information on superficial gas or liquid velocity is available, a detailed comparison between the correlation and the data is impossible. Since the data are so scatter, it may be difficult to make a quantitative discussion. However, the newly developed drift–flux correlation can give overall trend of the data satisfactorily. The gas velocities for a sinter injection seem to be slightly higher than those for a nozzle injection. This would be due to the enhancement of the liquid recirculation by the nozzle injection.



Fig. 9. Comparison of newly developed drift–flux correlation for a large diameter pipe without a horizontal section with nitrogen–water data taken in a pipe with an inner diameter of 0.102 m without a horizontal section [21].



Fig. 10. Comparison of newly developed drift–flux correlation for a large diameter pipe without a horizontal section with air– water data taken by Hills in a pipe with an inner diameter of 0.150 m [7].



Fig. 11. Comparison of newly developed drift–flux correlation for a large diameter pipe without a horizontal section with air– water data taken by Hashemi et al. in a pipe with an inner diameter of 0.305 m [16].

It should be noted here that the application of the newly developed drift–flux model to two-phase flow in a large diameter pipe at extremely small pipe length-to-diameter ratio such as  $z/D \leq 4$  should be examined by experimental data for  $z/D \leq 4$  to be taken in a future study. As suggested by the observation of Yoneda et al. [20], the flow would reach a quasi-developed state within relatively small  $z/D$  such as  $z/D \le 4$ . The effect of the flow



Fig. 12. Comparison of newly developed drift–flux correlation for a large diameter pipe without a horizontal section with air– water data taken by Ohnuki and Akimoto in a pipe with an inner diameter of 0.480 m [17].

development on the prediction accuracy of the newly developed drift–flux model is examined by the nitrogen– water data ( $D = 10.2$  cm) measured at three axial locations,  $z/D = 12.8$ , 26.6, and 41.8 [21]. The prediction errors for  $z/D = 12.8$ , 26.6, and 41.8 are estimated to be  $\pm 6.05\%$ , 4.44%, and 4.40%, respectively. Thus, since the newly developed drift–flux model has been validated in the region of  $4.2 \le z/D \le 108$ , it would give a good prediction even for developing flow except for extremely small pipe length-to-diameter ratio.

Next, the newly developed drift–flux correlation for the inlet flow regime such as cap or slug bubbles, Eq.  $(27)$  with Eqs.  $(22)$ – $(24)$ , and  $(33)$ , is compared with data taken by Hibiki and Ishii [22] in Fig. 13. Solid, broken, dotted and chain lines in Fig. 13 indicate calculated values by the newly developed drift–flux correlation, Ishiis correlation for slug flow, Kataoka–Ishiis correlation [14], and the correlation of Hirao et al. [11,12], respectively. The newly developed drift–flux correlation can predict the proper trend and the value of the experimental data very well. The newly developed drift– flux correlation also agrees with the correlation of Hirao et al., which suggests that the newly developed correlation may be also applicable to relatively high pressure steam–water flow system. An average relative deviation between the newly developed drift–flux correlation and 12 data sets shown in Fig. 13 is estimated to be  $\pm 3.04\%$ .

Thus, the newly developed drift–flux correlations give reasonably good predictions for the low-flow-data taken under various experimental conditions such as flow channel diameters (0.102–0.480 m), pressures (0.1–1.5 MPa), mixture volumetric fluxes (0.03–6.1 m/s), bubble injection methods (test pipe with or without a horizontal



Fig. 13. Comparison of newly developed drift–flux correlation for a large diameter pipe with a horizontal section with nitrogen–water data taken in a pipe with an inner diameter of 0.102 m with a horizontal section [22].

section), and fluid systems (air–water, nitrogen–water, and steam–water).

#### 3.5. Recommendation for drift–flux correlation in a large diameter pipe at high flow rate

In this section, the drift–flux correlations for upward two-phase flow in a large diameter pipe at high flow rate are recommended based on existing experimental data. Fig. 14 shows the comparison of various drift–flux correlations with air–water data taken by Hills in a large diameter pipe with an inner diameter of 0.150 m [7]. Fig. 15 also shows the comparison of various drift–flux correlations with the steam–water data taken by Hirao et al. in a large diameter pipe with an inner diameter of 0.102 m [11,12]. At high mixture volumetric flux, the magnitude of the local slip effect is much smaller than that of the distribution parameter effect, namely  $V_{\text{gi}}^+ \ll C_0 \langle j^+ \rangle$ . Thus, the distribution parameter is the dominant factor in the drift–flux model at high mixture volumetric flux. The drift–flux correlations tested in Figs. 14 and 15 give similar predictions, which agree with the data very well. This indicates that the distribution parameter given by Ishii (Eq. (5)) can be applicable to upward two-phase flow in a large diameter pipe over wide range of the mixture volumetric flux. At high mixture volumetric flux, the liquid recirculation may not be significant. Therefore, the drift-velocity correlations given by Ishii [4] may be applicable to upward two-phase flow in a large diameter pipe at high mixture volumetric flux, since they were derived based on rigorous drag law in two-phase flow. The drift–flux correlations in a large



Fig. 14. Comparison of various drift–flux correlations with air– water data taken by Hills in a pipe with an inner diameter of 0.150 m [7].

diameter pipe, which have been developed and recommended in this study, are summarized in Table 3.

# 3.6. Recommendations for future development of detailed drift–flux correlation in a large diameter pipe

In this study, the approximated drift–flux correlations for upward two-phase flow in a large diameter pipe have been developed and validated over wide flow and loop conditions. Since the correlations for the distribu-



Fig. 15. Comparison of various drift–flux correlations with steam–water data taken by Hirao et al. in a pipe with an inner diameter of 0.102 m [11,12].

tion parameter and the drift velocity in a large diameter pipe developed at low flow rate have not been validated separately by detailed local flow data, they should not be used individually. In a future study, detailed local measurements of flow parameters for gas and liquid phases in a large diameter pipe are recommended to develop a detailed and more rigorous drift–flux correlation taking account of the detailed flow structure. In addition to this, the drift–flux model for boiling flow in a large diameter pipe should be addressed in a future study. For a flow with generation of void at the wall due to nucleate boiling, the drift–flux model developed in this study may not give a good prediction. For such a flow condition, the distribution parameter should have a near-zero value at the beginning of the two-phase-flow region. With the increase in the cross-sectional mean void fraction, the peak of the local void fraction moves from the near-wall region to the central region. This will lead to the increase in the value of the distribution parameter as the void profile develops. In view of the basic characteristics described above, Ishii proposed the following simple correlation for boiling flow [4].

$$
C_0 = \left\{ C_{\infty} - (C_{\infty} - 1) \sqrt{\rho_g/\rho_f} \right\} \{ 1 - \exp(k \langle \alpha \rangle) \}, \qquad (35)
$$

where k is a coefficient. Ishii determined  $C_{\infty}$  to be 1.2 and the coefficient to be  $-18$  with experimental data taken in relatively small channels [4]. For a wall nucleate boiling in a large diameter pipe, the above model would also be sound, but the applicability of  $C_{\infty}$  correlation developed in this study and the coefficient determined for a small diameter pipe to two-phase flow in a large diameter pipe should be evaluated by boiling flow data in a large diameter pipe to be taken in a future study.

## 4. Conclusions

In view of the practical importance of the drift–flux model for two-phase-flow analysis in general and in the analysis of nuclear-reactor transients and accidents in particular, the distribution parameter and the drift velocity have been studied for upward two-phase flow in a large diameter pipe. The obtained results are as follows:

- (1) Existing analytical and experimental studies related to two-phase flow in a large diameter pipe were extensively reviewed.
- (2) It was shown that the two-phase flow characteristics in a large diameter pipe at low flow rate could be quite different from those in a relatively small diameter pipe. It revealed that the formation of cap bubbles and the occurrence of liquid recirculations would increase the distribution parameter and the drift velocity at low flow rate.





- (3) It was shown that the two-phase-flow characteristics in a large diameter pipe at low flow rate could be influenced by the inlet flow regime significantly. Two types of drift–flux correlations were developed for two typical inlet flow regimes such as (i) uniformly dispersed bubbly flow regime and (ii) cap bubbly or slug flow regime.
- (4) The newly developed drift–flux correlations gave reasonably good predictions for the low-flow-data taken under various experimental conditions such as flow channel diameters (0.102–0.480 m), pipe length-to-diameter ratio (4.2–108), pressures (0.1– 1.5 MPa), mixture volumetric fluxes (0.03–6.1 m/s), bubble injection methods (test pipe with or without a horizontal section), and fluid systems (air–water, nitrogen–water, and steam–water).
- (5) Drift–flux correlations developed by Kataoka–Ishii, and Ishii were recommended for cap bubbly flow, and churn and annular flows, respectively, based on the comparisons of the correlations with existing experimental data.

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